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ANALYTICAL SOLUTIONS OF A SET OF EQUATIONS OF HEAT AND MASS TRANSFER FOR A SEMI-BOUNDED MEDIUM WITH DIFFERENT BOUNDARY CONDITIONS

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Аннотация—В докладе сформулированы две задачи из аналитической теории переноса тепла и массы. Аналитические решения даны с помощью комплексного применения интегральных преобразований Фурье и Лапласа.

NOMENCLATURE

α, β, γ,	variables of integration over space co-ordinates;
τ,	variable of integration over time;
x, y, z,	co-ordinates of a point in a three-dimensional space;
t,	time;
ξ, η, ζ,	transformed co-ordinates x, y, z in an image region;
P,	parameter of the Laplace integral transformation over time t;
$\doteqdot$,	sign of transition from an original to an image.

THE present paper deals with an analytical theory of heat and mass transfer of a bound matter for a semi-bounded medium with boundary conditions of the first and second types.

In Luikov's papers the analytical theory of internal heat and mass transfer in capillary porous bodies is systematically developed, and is described by a set of partial differential equations of the parabolic type. This set in potential form may be represented as follows:

$$\left.\begin{aligned}\frac{\partial u}{\partial t} &= a_1^2\nabla^2 u + a_2^2\nabla^2 T,\\ \frac{\partial T}{\partial t} &= a_3^2\nabla^2 T + a_4\frac{\partial u}{\partial t},\end{aligned}\right\} \quad (1)$$

where

$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} = \Delta$—is the Laplace operator;

$u(x, y, z, t)$—is the distribution function of a bound matter (moisture), kg/kg;

$T(x, y, z, t)$—is the potential function of temperature distribution, deg;

a_1^2—is the conductivity potential coefficient of the mass of bound matter, m²/hr;

a_3^2—is the heat conductivity potential coefficient (thermal diffusivity coefficient) m²/hr, $a_2^2 = a_1^2\delta$;

δ—is the Soret coefficient, deg⁻¹.

Many problems in the analytical theory of heat and mass transfer phenomenon of bound matter are reduced to the solution of the set of equations (1) for different boundary conditions. Therefore the accumulation of solutions of system (1) at different initial and boundary conditions has considerable theoretical and practical interest.

Solutions for system (1) at boundary conditions of the first and second kinds for a semi-bounded one-dimensional and multi-dimensional medium were found by the author. These solutions are given below.

Problem 1. Find functions $u(x, y, z, t)$, $T(x, y, z, t)$ in the region

$$\Omega \begin{cases} 0 \leqslant x < \infty \\ -\infty < y < \infty \quad t > 0, \\ -\infty < z < \infty \end{cases}$$

satisfying system (1) and boundary conditions of the second kind:

$$u(x, y, z, 0) = f_1(x, y, z), \quad T(x, y, z, 0) = f_2(x, y, z); \tag{2}$$

$$\left.\frac{\partial u}{\partial x}\right|_{x=0} = \varphi_1(y, z, t), \quad \left.\frac{\partial T}{\partial x}\right|_{x=0} = \varphi_2(y, z, t). \tag{3}$$

Problem 2. Solve system (1) in the region of Ω with boundary conditions of the first kind:

$$u(x, y, z, 0) = f_1(x, y, z), \quad T(x, y, z, 0) = f_2(x, y, z); \tag{4}$$

$$u(0, y, z, t) = \psi_1(y, z, t), \quad T(0, y, z, t) = \psi_2(y, z, t). \tag{5}$$

Functions $u(x, y, z, t)$, $T(x, y, z, t)$ are sought in the class for which the Fourier transformations in space co-ordinates and the Laplace transformation in time, t, in the region of Ω are applicable. We start from the regularity of the functions being determined at infinity $u(x, y, z, t) \to 0$, $T(x, y, z, t) \to 0$ at $r^2 = x^2 + y^2 + z^2 \to \infty$.

The application of integral transformation methods in solving system (1) makes it possible to divide effectively the functions u and T and to carry out the solution of the problem to completion.

Let

$$\frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} u(x, y, z, t) \exp[i(\eta y + \zeta z)]\, dy\, dz = v(x, \eta, \zeta, t),$$

$$\frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} T(x, y, z, t) \exp[i(\eta y + \zeta z)]\, dy\, dz = T(x, \eta, \zeta, t),$$

then

$$\frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \left(\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) \exp[i(\eta y + \zeta z)]\, dy\, dz = -(\eta^2 + \zeta^2)\, v(x, \eta, \zeta, t),$$

$$\frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \left(\frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right) \exp[i(\eta y + \zeta z)]\, dy\, dz = -(\eta^2 + \zeta^2)\, T(x, \eta, \zeta, t).$$

Denote

$$\sqrt{\left(\frac{2}{\pi}\right)}\int_0^{\infty} v(x, \eta, \zeta, t) \cos x\xi\, dx = v^*(\xi, \eta, \zeta, t),$$

$$\sqrt{\left(\frac{2}{\pi}\right)}\int_0^{\infty} T(x, \eta, \zeta, t) \cos x\,\xi\, dx = T^*(\xi, \eta, \zeta, t).$$

Applying the cosine transformation to the second derivatives

$$\frac{\partial^2 v(x, \eta, \zeta, t)}{\partial x^2}, \quad \frac{\partial^2 T(x, \eta, \zeta, t)}{\partial x^2}$$

with regard to transformed boundary conditions (3)

$$\Phi_k(\eta, \zeta, t) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \varphi_k(y, z, t) \exp [i(\eta y + \zeta z)] \, dy \, dz \quad (k = 1, 2) \tag{3*}$$

we obtain

$$\sqrt{\left(\frac{2}{\pi}\right)} \int_0^\infty \frac{\partial^2 v}{\partial x^2} \cos x\,\xi \, dx = - \xi^2 v^* (\xi, \eta, \zeta, t) - \sqrt{\left(\frac{2}{\pi}\right)}\, \Phi_1(\eta, \zeta, t),$$

$$\sqrt{\left(\frac{2}{\pi}\right)} \int_0^\infty \frac{\partial^2 T}{\partial x^2} \cos x\,\xi \, dx = - \xi^2 T^*(\xi, \eta, \zeta, t) - \sqrt{\left(\frac{2}{\pi}\right)}\, \Phi_2(\eta, \zeta, t).$$

Apply to system (1) the bilateral integral Fourier transformation in variables, y, z, and the cosine transformation in the variable, x. Then, taking into account condition (3*) and the regularity of functions at infinity, we obtain:

$$\left.\begin{aligned} \frac{\partial v^*}{\partial t} &= - a_1^2(\xi^2 + \eta^2 + \zeta^2)v^* - a_2^2(\xi^2 + \eta^2 + \zeta^2)T^* - \sqrt{\left(\frac{2}{\pi}\right)}\,(a_1^2\Phi_1 + a_2^2\Phi_2), \\ \frac{\partial T^*}{\partial t} &= - a_3^2(\xi^2 + \eta^2 + \zeta^2)T^* + a_4 \frac{\partial v^*}{\partial t} - a_3^2 \sqrt{\left(\frac{2}{\pi}\right)}\, \Phi_2 \end{aligned}\right\} \tag{1*}$$

For the system (1*) the initial conditions will be:

$$F^*(\xi, \eta, \zeta) = \sqrt{\left(\frac{2}{\pi}\right)} \int_0^\infty F_k(x, \eta, \zeta) \cos x\,\xi \, dx \ldots, \tag{2*}$$

where

$$F_k(x, \eta, \zeta) = \sqrt{\left(\frac{2}{\pi}\right)} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_k(x, y, z) \exp [i(\eta y + \zeta z)] \, dy \, dz.$$

Assume

$$\bar{v}^*(\zeta, \eta, \xi, P) = \textstyle\int_0^\infty v^*(\xi, \eta, \zeta, t) \exp (-Pt) \, dt,$$

$$\bar{T}^*(\xi, \eta, \zeta, P) = \textstyle\int_0^\infty T^*(\xi, \eta, \zeta, t) \exp (-Pt) \, dt.$$

Then after the Laplace integral transformation system (1*) with due regard for the initial conditions (2*) will be as follows:

$$\left.\begin{aligned} &(P + a_1^2\rho^2)\bar{v}^*(\xi, \eta, \zeta, P) + a_2^2\rho^2\bar{T}^*(\xi, \eta, \zeta, P) \\ &\qquad = - \sqrt{\left(\frac{2}{\pi}\right)}\, [a_1^2\bar{\Phi}_1(\eta, \zeta, P) + a_2^2\bar{\Phi}_2(\eta, \zeta, P)] + F_1^*(\xi, \eta, \zeta), \\ &- a_4P\bar{v}^*(\xi, \eta, \zeta, P) + (a_3^2\rho^2 + P)\bar{T}^*(\xi, \eta, \zeta, P) \\ &\qquad = - \sqrt{\left(\frac{2}{\pi}\right)}\, a_3^2\bar{\Phi}_2(\eta, \zeta, P) - a_4F_1^*(\xi, \eta, \zeta) + F_2^*(\xi, \eta, \zeta), \\ &\qquad (\rho^2 = \xi^2 + \eta^2 + \zeta^2) \end{aligned}\right\} \tag{1**}$$

$$D(\xi, \eta, \zeta, P) = \begin{vmatrix} (P + a_1^2\rho^2) & a_2\rho_2 \\ -a_4P & (P + a_3^2\rho^2) \end{vmatrix} = P^2 + P(a_1^2 + a_3^2 + a_2^2a_4)\rho^2 + a_1^2a_3^2\rho^4$$

Denote by $P_1 = -a\rho^2$, $P_2 = -b\rho^2$ the roots of the determinant $D(\xi, \eta, \zeta, P)$ with respect to the parameter, P.

The analysis of these roots shows that the values a, b are positive. This means that we may assume $a = c_1^2$; $b = c_2^2$.

From system (1**) the transforms of potential distributions of mass and temperature may be written in the form:

$$\bar{v}^*(\xi, \eta, \zeta, P) = \frac{D_1(\xi, \eta, \zeta, P)}{D(\xi, \eta, \zeta, P)}, \quad \bar{T}^*(\xi, \eta, \zeta, P) = \frac{D_2(\xi, \eta, \zeta, P)}{D(\xi, \eta, \zeta, P)}.$$

$$D_1(\xi, \eta, \zeta, P) = \begin{vmatrix} -\sqrt{\left(\frac{2}{\pi}\right)}(a_1^2\bar{\Phi}_1 + a_2^2\bar{\Phi}_2) + F_1^* & a_2^2\rho^2 \\ -a_3^2\sqrt{\left(\frac{2}{\pi}\right)}\bar{\Phi}_2 + F_2^* - a_4F_1^* & (P + a_3^2\rho) \end{vmatrix},$$

$$D_2(\xi, \eta, \zeta, P) = \begin{vmatrix} (P + a_1^2\rho^2) & -\sqrt{\left(\frac{2}{\pi}\right)}(a_1^2\bar{\Phi}_1 + a_2^2\bar{\Phi}_2) + F_1^* \\ -a_4P & -a^2\sqrt{\left(\frac{2}{\pi}\right)}\bar{\Phi}_2 + F_2^* - a_4F_1^* \end{vmatrix}.$$

The transform of mass distribution of a bound substance in the expanded form will be,

$$\bar{v}^*(\xi, \eta, \zeta, P) = \frac{-(P + a_3^2\rho^2)\sqrt{\left(\frac{2}{\pi}\right)}\bar{\Phi}(\eta, \zeta, P)}{D(\xi, \eta, \zeta, P)} + \frac{a_3^2a_2^2\sqrt{\left(\frac{2}{\pi}\right)}\bar{\Phi}_2(\eta, \zeta, P)}{D(\xi, \eta, \zeta, P)} + \frac{(P + a_3^2\rho^2)F_1^*(\xi, \eta, \zeta, P) + \rho^2F^*(\xi, \eta, \zeta)}{D(\xi, \eta, \zeta, P)},$$

where

$$\bar{\Phi}(\eta, \zeta, P) = a_1^2\bar{\Phi}_1 + a_2^2\bar{\Phi}_2, \quad F^*(\xi, \eta, \zeta) = a_2^2(a_4F_1^* - F_2^*).$$

For the parameter transition from the image to the original by the parameter t, we use the formula for the determination of the original when the image is a normal fractional-rational function with poles of the first order:

$$\frac{Q_m(P)}{R_n(P)} = \sum_{k=1}^{n} \frac{Q_m(P_k)}{R_n'(P_k)} \exp(P_kt), \tag{+}$$

where P_k are simple roots of integral function $R_n(P)$, $n > m$.

Using this formula and the corresponding theorem on convolutions we, in our case, obtain:

$$v^*(\xi, \eta, \zeta, t) = \sum_{k=1}^{2} \alpha_k \left\{\sqrt{\left(\frac{2}{\pi}\right)}\int_0^t \left[\frac{a_3^2 \cdot a_2^2}{a_3^2 - c_k^2}\Phi_2(\eta, \zeta, \tau) - \Phi(\eta, \zeta, \tau)\right] \times \exp[-\rho^2c_k^2(t-\tau)]\,d\tau + \left[F_1^*(\xi, \eta, \zeta) + \frac{F^*(\xi, \eta, \zeta)}{a_3^2 - c_2^k}\right]\exp(-\rho^2c_k^2t)\right\},$$

where

$$\alpha_k = \frac{a_3^2 - c_k^2}{a_1^2 + a_3^2 + a_2^2a_4 - 2c_k^2}.$$

Subject the function $v^*(\xi, \eta, \zeta, t)$ in variable, ξ, to an inverse Fourier cosine-transformation. Then we get:

$$v(x, \eta, \zeta, t) = \sum_{k=1}^{2} \frac{2a_k}{\pi} \left\{ \int_0^t \left[\frac{a_2^2 a_3^2}{a^2 - c_k^2} \Phi_2(\eta, \zeta, \tau) - \Phi(\eta, \zeta, \tau) \right] \exp\left[-(\eta^2 + \zeta^2)(t - \tau) c_k^2\right] d\tau \right.$$

$$\times \int_0^\infty \exp\left[-\zeta^2 c_k^2 (t - \tau) \cos x\zeta \, d\zeta\right] + \int_0^\infty \left[F_1(a, \eta, \zeta) + \frac{F(a, \eta, \zeta)}{a_3^2 - c_k^2} \right] \exp\left[-c_k^2(\eta^2 + \zeta^2)t\right] . da$$

$$\left. \times \int_0^\infty \exp(-\zeta^2 . c_k^2 t) \cos a\zeta \cos x\zeta . d\zeta \right\}.$$

Using the value of the improper integral

$$\int_0^\infty \exp(-a^2 x^2) \cos(2bx) . dx = \frac{\sqrt{\pi}}{2a} \exp\left(-\frac{b^2}{a^2}\right)$$

and the equality $2 \cos a\zeta . \cos \zeta = \cos \zeta(x + a) + \cos \zeta(x - a)$ we write:

$$v(x, \eta, \zeta, t) = \sum_{k=1}^{2} \frac{a_k}{c_k\sqrt{\pi}} \left\{ \int_0^t \left[\frac{a_3^2 . a_2^2}{a_3^2 - c_k^2} \Phi_2(\eta, \zeta, \tau) - \Phi(\eta, \zeta, \tau) \right] \right.$$

$$\times \frac{\exp\left[-(\eta^2 + \zeta^2)(t - \tau)c_k^2\right]}{\sqrt{(t - \tau)}} \cdot \exp\left[\frac{x^2}{4c_k^2(t - \tau)}\right] d\tau + \frac{1}{2\sqrt{t}} \int_0^\infty \left[F_1(a, \eta, \zeta) \right.$$

$$\left. \left. + \frac{F(a, \eta, \zeta)}{a_3^2 - c_k^2} \right] \exp\left[-c_k^2(\eta^2 + \zeta^2)t\right] \left[\exp\left(-\frac{(x - a)^2}{4c_k^2 t}\right) + \exp\left(-\frac{(x + a)^2}{4c_k^2 t}\right) \right] da \right\}.$$

For the transition to the original in the remaining co-ordinates we use the conversion formula for a two-dimensional Fourier transformation

$$\exp\left[-(\eta^2 + \zeta^2)a^2 t\right] \doteqdot \frac{1}{2a^2 t} \exp\left(\frac{y^2 + z^2}{4a^2 t}\right).$$

Thus after a reverse conversion through variables η, ζ and application of theorems on convolutions the function of mass distribution of bound matter will have the form:

$$u(x, y, z, t) = \sum_{k=1}^{2} a_k \left\{ \frac{1}{4(c_k\sqrt{\pi})^3} \int_0^t \int_{-\infty}^\infty \int_{-\infty}^\infty \frac{\varphi_k^*(\beta, \gamma, \tau)}{\sqrt{[(t - \tau)^3]}} \exp\left[-\frac{x^2 + (y - \beta)^2 + (z - \gamma)^2}{4c_k^2(t - \tau)}\right] d\tau \, d\beta \, d\gamma \right.$$

$$+ \frac{1}{[2\sqrt{(\pi t)}c_k]^3} \int_0^\infty \int_{-\infty}^\infty \int_{-\infty}^\infty f_k^*(a, \beta, \gamma) \exp\left[-\frac{(x - a)^2 + (y - \beta)^2 + (z - \gamma)^2}{4c_k^2 t}\right]$$

$$\left. \times \left[1 + \exp\left(-\frac{ax}{c_k^2 t}\right)\right] da \, d\beta \, d\gamma \right\}. \quad (6)$$

In formula (6) the following notation is adopted

$$\varphi_k^*(y, z, t) = \frac{a_2^2 c_k}{a_3^2 - c_k^2} \varphi_2(y, z, t) - a_1^2 \varphi_1(y, z, t),$$

$$f_k^*(x, y, z) = \left(1 + \frac{a_2^2 a_4}{a_3^2 - c_k^2}\right) f_1(x, y, z) - \frac{a_2^2}{a_3^2 - c_k^2} f_2(x, y, z).$$

The transition from the image of the temperature distribution function to the original is effected by the same successive operations which were carried out in the determination of the distribution function of moisture. On this basis omitting these transformations we may write down the final result:

$$\left.\begin{aligned} T(x, y, z, t) = \sum_{k=1}^{2} \gamma_k \Bigg\{ \frac{1}{4[\sqrt{(\pi)} c_k]^3} \int_0^t \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{\varphi_k^{**}(\beta, \gamma, \tau)}{\sqrt{(t-\tau)^3}} \exp\left[\frac{x^2 + (y-\beta)^2 + (z-\gamma)^2}{4c_k^2(t-\tau)}\right] \\ \times d\tau\, d\beta\, d\gamma + \frac{1}{[2\sqrt{(\pi t)} c_k]^3} \int_0^{\infty} \int_{-\infty}^{-\infty} \int_{-\infty}^{\infty} f_k^{**}(\alpha, \beta, \gamma) \exp\left[-\frac{(x-\alpha)^2 + (y-\beta)^2 + (z-\gamma)^2}{4c_k^2 t}\right] \\ \times \left[1 + \exp\left(-\frac{\alpha x}{c_k^2 t}\right)\right] d\alpha\, d\beta\, d\gamma \Bigg\}; \end{aligned}\right\} \quad (7)$$

where

$$\varphi_k^{**}(y, z, t) = \left[\frac{a_2^2 a_4 \cdot c_k^2}{a_1^2 - c_k^2} - a_3^2\right] \varphi_2(y, z, t) + \frac{a_4 a_1^2 c_k^2}{a_1^2 - c_k^2} \varphi_1(y, z, t);$$

$$f_k^{**}(x, y, z) = f_2(x, y, z) - \frac{a_1^2 a_4}{a_1^2 - c_k^2} f_1(x, y, z);$$

$$\gamma_k = \frac{a_1^2 - c_k^2}{a_1^2 + a_3^2 + a_2^2 a_4 - 2c_k^2}.$$

Formulas (6) and (7) represent a solution to the first problem with very general boundary conditions.

It is known from the theory of a non-stationary heat and mass transfer process that the Soret coefficient has a very small value, so that the second term in system (1) may in some cases be neglected. When $a_2^2 = 0$ formulas (6) and (7) after certain simplifications will be reduced to:

$$\left.\begin{aligned} u_0(x, y, z, t) = \frac{1}{[2\sqrt{(\pi t)} a_1]^3} \int_0^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f_1(\alpha, \beta, \gamma) \exp\left[\frac{(x-\alpha)^2 + (y-\beta)^2 + (z-\gamma)^2}{4a_1^2 t}\right] \\ \times \left[1 + \exp\left(-\frac{\alpha x}{a_1^2 t}\right)\right] d\alpha\, d\beta\, d\gamma - \frac{1}{4\sqrt{(\pi^3)} a_1} \int_0^t \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{\varphi_1(\rho, \gamma, t)}{\sqrt{[(t-\tau)^3]}} \\ \times \exp\left[-\frac{x^2 + (y-\beta)^2 + (z-\gamma)^2}{4a_2^2(t-\tau)}\right] d\tau\, d\beta\, d\gamma. \end{aligned}\right\} \quad (6^*)$$

$$\left.\begin{aligned} T_0(x, y, z, t) = \frac{1}{4a_3\sqrt{(\pi^3)}} \int_0^t \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{\varphi(\beta, \gamma, \tau)}{\sqrt{[(t-\tau)^3]}} \exp\left[-\frac{x^2 + (y-\beta)^2 + (z-\gamma)^2}{4a_3^2(t-\tau)}\right] d\tau\, d\beta\, d\gamma \\ - \frac{a_4 a_1}{4(a \quad - a^2)\sqrt{(\pi^3)}} \int_0^t \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{\varphi_1(\beta, \gamma, \tau)}{\sqrt{[(t-\tau)^3]}} \exp\left[-\frac{x^2 + (y-\beta)^2 + (z-\gamma)^2}{4a_3^2(t-\tau)}\right] d\tau\, d\beta\, d\gamma \\ + \frac{1}{[2a_3\sqrt{(\pi t)}]^3} \int_0^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(\alpha, \beta, \gamma) \exp\left[-\frac{(x-\alpha)^2 + (y-\beta)^2 + (z-\gamma)^2}{4a_3^2 t}\right] \\ \times \left[1 + \exp\left(-\frac{\alpha v}{a_3^2 t}\right)\right] d\alpha\, d\beta\, d\gamma + \frac{a_4}{a_4(a_1^2 - a_3^2)\,[2\sqrt{(\pi t)}]^3} \int_0^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f_1(\alpha, \beta, \gamma) \\ \times \exp\left[-\frac{(x-\alpha)^2 + (y-\beta)^2 + (z-\gamma)^2}{4a_1^2 t}\right] \times \left[1 + \exp\left(-\frac{\alpha x}{a_1^2 t}\right)\right] d\alpha\, d\beta\, d\gamma. \end{aligned}\right\} \quad (7^*)$$

where

$$\varphi(y, z, t) = \frac{a_4 a_1^2}{a_1^2 - a_3^2} \varphi_1(y, z, t) - \varphi_2(y, z, t);$$

$$f(x, y, z) = f_2(x, y, z) - \frac{a_1^2 a_4}{a_1^2 - a_3^2} f_1(x, y, z).$$

Formula (6*) represents the known classical solution of a thermal conductivity equation.

Consider a particular case of the given problem when the boundary conditions are constant values.

Let

$$f_1(x, y, z) = u_0 = \text{const.}, \qquad f_2 = (x, y, z) = \theta = \text{const.},$$

$$\varphi_1(y, z, t) = q_1 = \text{const.}, \qquad \varphi_2 = (y, z, t) = q_2 = \text{const.}$$

Then formula (6*) after some simplifications will be:

$$u_0(x, y, z, t) = u_0 + q_1 x \left[\operatorname{erf}(\alpha) - \frac{\exp(-\alpha^2)}{\alpha\sqrt{\pi}} \right] \ldots, \tag{8}$$

where

$$\alpha = \frac{x}{2a_1\sqrt{t}}; \quad \operatorname{erf}(\alpha) = 1 - \operatorname{erf}(\alpha) = 1 - \frac{2}{\sqrt{\pi}} \int_0^\alpha \exp(-z^2)\, dz.$$

Substituting the values u_0, θ, q_1, q_2 into formula (7*) we obtain:

$$\left.\begin{aligned} T_0(x, y, z, t) = \theta + q^* x \left[\frac{\exp(-\alpha_1^2)}{\alpha_1\sqrt{\pi}} - \operatorname{erf}(\alpha_1) \right] - \frac{a_4 \,.\, a_1^2 \,.\, q_1 \,.\, x}{a_1^2 - a_3} \left[\frac{\exp(-\alpha_3^2)}{\alpha_3\sqrt{\pi}} - \operatorname{erf}(\alpha_3) \right] \\ \alpha_1 = \frac{x}{2a_3\sqrt{t}}, \qquad \alpha_3 = \frac{x}{2a_3\sqrt{t}}, \qquad q^* = \left[\frac{a_1^2 a_4 q_1}{a_1^2 - a_3^2} - q_2 \right]. \end{aligned}\right\} \tag{9}$$

The solution of the second problem is carried out by the same methods as used in the first one.

The transition from the required functions $u(x, y, z, t)$, $T(x, y, z, t)$ to their images is effected by the following successive integral transformations: double integral Fourier transformation by variables y, z, sine-transformation by x and the Laplace transformation by time t. As a result of the application of these transformations to system (1) and to boundary conditions (4) and (5) we find the solution of a set of differential equations of internal heat and mass transfer in the images.

The transition from the image to the original is effected by reverse integral transformations They are carried out in inverse order.

Omitting all these transformations we may write down the result of the second problem considered:

$$\left.\begin{aligned} u(x, y, z, t) = \sum_{k=1}^{2} \frac{1}{[2\sqrt{(\pi)}c_k]^3} \Bigg\{ \frac{x}{c_k^2} \int_0^t \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{[m_k \psi_1(\beta, \gamma, \tau) - n_k \psi_2(\beta, \gamma, \tau)]}{\sqrt{[(t-\tau)^5]}} \\ \times \exp\left[-\frac{x^2 + (y-\beta)^2 + (z-\gamma)^2}{4c_k^2(t-\tau)} \right] d\tau\, d\beta\, d\gamma + \frac{1}{\sqrt{(t^3)}} \int_0^\infty \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \\ \times [\nu_k f_1(\alpha, \beta, \gamma) - \mu_k f_2(\alpha, \beta, \gamma)] \,.\, \exp\left[-\frac{(x-\alpha)^2 + (y-\beta)^2 + (z-\gamma)^2}{4c_k^2 t} \right] \\ \times \left[1 - \exp\left(-\frac{\alpha x}{c_k^2 t} \right) \right] d\alpha\, d\beta\, d\gamma \Bigg\}. \end{aligned}\right\} \tag{10}$$

$$
\left.\begin{aligned}
T(x, y, z, t) = \sum_{k=1}^{2} \frac{1}{[2\sqrt{(\pi)}c_k]^3} \left\{ \frac{x}{c_k^2} \int_0^t \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{[\beta_k \psi_2(\beta, \gamma, \tau) - \gamma_k \psi_1(\beta, \gamma, \tau)]}{\sqrt{[(t-\tau)^5]}} \right. \\
\times \exp\left[-\frac{x^2 + (y-\beta)^2 + (z-\gamma)^2}{4c_k^2(t-\tau)} \right] \mathrm{d}\tau\, \mathrm{d}\beta\, \mathrm{d}\gamma + \frac{1}{\sqrt{(t^3)}} \int_0^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} [\lambda_k f(\alpha, \beta, \gamma) - \chi_k f_2(\alpha, \beta, \gamma)] \\
\times \exp\left[-\frac{(x-\alpha)^2 + (y-\beta)^2 + (z-\gamma)^2}{4c_k^2 t} \right] \left[1 - \exp\left(\frac{\alpha x}{c_k^2 t} \right) \right] \mathrm{d}\alpha\, \mathrm{d}\beta\, \mathrm{d}\gamma \left. \vphantom{\int} \right\},
\end{aligned}\right\} \quad (11)
$$

where

$$
m_k = A_k a_1^2(a_3^2 - c_k^2); \quad n_k = A_k a_2^2 c_k^2; \quad \nu_k = 1 - A_k(a_1^2 - c_k^2); \quad \mu_k = A_k a_2^2;
$$
$$
\beta_k = A_k[(a_1^2 - c_k^2)a_3^2 - a_2^2 a_4 c_k^2]; \quad \gamma_k = A_k a_1^2 a_4 c_k^2; \quad \lambda_k = A_k(a_1^2 - c_k^2);
$$
$$
\chi_k = a_1^2 a_4 A_k; \quad A_k = (a_1^2 + a_3^2 + a_2^2 a_4 - 2c_k^2)^{-1}.
$$

It should be noted that for the transition from the image to the original by parameter ξ it is necessary to use the known formula of the improper integral

$$
\int_0^{\infty} \xi \exp(-a^2 \xi^2 t) \sin x\xi \, \mathrm{d}\xi = \frac{x\sqrt{\pi}}{4a^3\sqrt{(t^3)}} \exp\left(-\frac{x^2}{4a^2 t} \right). \quad (12)
$$

With the help of a similar method the author has solved boundary problems of the first and the second kind for a system of "n" non-uniform differential equations of a parabolic type.

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Abstract—Two problems for the system of equations of heat and mass transfer are considered. The Laplace-Fourier integral transformations are used for the solution.

Résumé—Deux cas de système d'équations de transport de chaleur et de masse sont considérés. Pour les résoudre on utilise les transformations d'intégrales de Fourier et de Laplace.

Zusammenfassung—Die Gleichungssysteme für Wärme- und Stoffübertragung werden auf zwei Problemstellungen angewandt. Zur Lösung dienten die Laplace-Fourier-Integraltransformationen.